Expecting the Unexpected: Surprises on the Hunt for NonArchimedean Fractals

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The Metric Fractal Machine

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Motivating the Hunt!

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3. Application: generalize fractal theory, and provide strictly non-metrizable fractals

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Committing to the Hunt

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Let F be an ordered field. An F-metric space is a set X together with a function $d : X \times X \to F^{\geq 0}$, satisfying the usual metric space axioms, but with F in place of $\mathbb{R}$.

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1. For all $x \in X$ and $r \in R$, $x \in \beta(x, r)$
2. Every $r \in R$ has a swing value – an $s \in R$ such that, if $x \in \beta(y, s)$, then $\beta(y, s) \subset \beta(x, r)$

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- What maps are contractions?
- What conditions do we need to guarantee that contractions have unique fixed points?

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  1. Maps $f : X \to X$ where there is an $r \in [0, 1)$ such that $\rho(f(x), f(y)) \leq r \cdot \rho(x, y)$

Completeness: a Grim Truth

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A net is a collection of points, indexed on a a directed set.

- The broader notions of Cauchy and complete fall out immediately.

The Completeness We Deserve

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  3. It isn't necessarily true that a hyperspace inherits spherical completeness

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The idea here is that we need to consider sequences (or nets) that are Cauchy with respect to a certain measuring stick. This measuring stick is given by the "swing net", $(r_k)_{k \in I}$, where $j < k$ implies that r_k is a swing value for r_j.

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- Every space has a natural level completion.
- There is a very nice characterization of level complete ordered fields

Getting to the Fixed Point

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Conjecture: we only need level completeness for this theorem to be true.

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- Uniform topology is also the wrong perspective for contractions.

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- Uniform spaces are wholly unsuitable for contractions

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- The condition of completeness is suspicious
- What sets in $\mathcal{L}$ are totally bounded?

Surprise #4!

- In $\mathcal{L}$, compact sets are always countable.

First it Gets Worse

Theorem

In any fully nonarchimedean space, compact sets are countable.

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Theorem

A space is level compact if and only if it is level complete and level bounded.

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